As a particular case of (7) we have

$$R = Ae^{m_1t} + Be^{m_2t}, \tag{8}$$

where A and B are arbitrary constants, since a pair of values of a and α can be found such that m_1 and m_2 satisfy (6). If (8) is specialized by setting A and B equal, and taking a pair of conjugate complex numbers for m_1 and m_2, we obtain:

$$R = Ce^{mt} \cos nt, \tag{9}$$

which defines a logarithmic spiral when $n = 0$, and a cycloidal curve when $m = 0$, the cases obtained by Professor Light.

It is interesting to note that (6) has at most two real roots (since the equation obtained from it by differentiating both sides with respect to m evidently has at most one real root), but an infinity of complex roots.

Since the Cartesian equation of a curve given in the form (1) would be obtained by integrating the equations:

$$\begin{aligned} dx &= ds \cos t = R \cos t\, dt = f(t) \cos t\, dt, \\ dy &= ds \sin t = R \sin t\, dt = f(t) \sin t\, dt, \end{aligned} \tag{10}$$

and eliminating the parameter t, we see that the curve corresponding to (7) could be obtained from the curves corresponding to the separate terms of the sum by locating the points whose abscissas are the sums of the abscissas and whose ordinates are the sums of the ordinates of points on the component curves whose tangents are parallel, *i.e.*, points corresponding to the same value of t.

The problem admits of several generalizations. Puiseux[1] extended his solution to the case where it is merely required to find curves whose nth evolutes are similar to themselves. This extension presents no new difficulties. Binet[1] studied surfaces such that the locus of one of the two centers of curvature at each point was a surface equal to the original surface. We might also inquire whether there are any twisted curves such that one of their evolutes is similar; or such that the locus of centers of osculating spheres or of centers of curvature gives similar curves. This question is more difficult than that for the plane, owing to the greater number of constants determining a displacement, and the writer knows no successful method of attacking it, nor whether any particular solutions besides certain circular helices are known.

II. Geometric Proofs of the Law of Tangents.[2]

By R. M. Mathews, Wesleyan University.

Professor Lovitt's sixth proof, which uses the circumscribed circle, is given by: Killing und Hovestadt, *Handbuch des Mathematischen Unterrichts*, vol. 2, p. 27.

In *School Science and Mathematics*, vol. 15, pp. 798–801, I published an article, "Proofs of the Law of Tangents," in which I gave five different proofs.

[1] L. c.

[2] Extract from a letter to the editor.

The proofs of Hobson, Wilczynski, Hall and Frink, Paterson (*Elementary Trigonometry*), and Killing and Hovestadt were reduced to a common notation. This article called forth two notes, one from Professor E. R. Hedrick, *School Science and Mathematics*, vol. 16, pp. 347–348, containing a proof of his own which is used by his permission in Kenyon and Ingold, *Plane and Spherical Trigonometry*; and one from Professor C. N. Mills (*loc. cit.*, p. 607), who gives a proof found in "an old text-book of trigonometry."

Professor Lovitt begins by constructing segments for $a + b$ and $a - b$ and then discovering angles equal to $\frac{1}{2}(A + B)$ and $\frac{1}{2}(A - B)$. We can also begin by constructing these angles and then discovering the segments. In this connection I gave in the article cited a proof of my own that I have not seen anywhere in print.

Take $a > b$; let XCY be the bisector of the exterior angle at C, so that $\angle ACY = \frac{1}{2}(A + B)$. Draw $AG \parallel XY$; $AF \perp XY$; $BGD \perp XY$. Then $\angle BAG = \frac{1}{2}(A - B)$.

$$\tan \tfrac{1}{2}(A - B) = \frac{BG}{GA} = \frac{BD - GD}{DC + CF} = \frac{(a - b) \sin \frac{1}{2}(A + B)}{(a + b) \cos \frac{1}{2}(A + B)} = \frac{a - b}{a + b} \tan \tfrac{1}{2}(A + B).$$

By adding a few lines to Professor Lovitt's figure I have been able to reduce all the proofs at hand to one figure. From this figure so many other possible proofs appear and the temptation to add still other lines is so great that it seems the prudent thing to rest content with what we have.

III. Expository Papers for the Association.[1]

By E. J. Wilczynski, University of Chicago.

The Mathematical Association of America has reached the conclusion that it can assist very effectively in enlarging the mental horizon of its members by presenting, from time to time, properly conceived papers of an expository character. But the question immediately arises: what is meant by an expository paper, and what are the most desirable characteristics of such a paper? It is my purpose to answer this question very briefly and in a preliminary fashion. Later, when as we hope, a large number of successful expository papers will be available for analysis and comparison, it may become possible to answer this question far more fully.

1. *Choice of Subject.* We all observe, from time to time, in connection with our work of teaching and research, that certain subjects are either omitted entirely from our textbooks, or are treated in inadequate fashion. If it is a subject of general interest and importance which has been thus slighted, it clearly offers a desirable field for an expository paper. Or else we may be interested in some advanced work, and the idea may come to us to explain this work to a non-technical audience. We should follow such impulses, especially when they are

[1] An address delivered before the Mathematical Association of America, and Section L of the American Association for the Advancement of Science at the University of Chicago, December 28, 1920.